Modelling Elastic Collisions Process

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Abstract

The collision of two wagons is used to introduce several modelling concepts, to high school students. This problem is usually treated only by the conservation laws of momentum and energy with respect to their values before and after the collision. Our model uses springs to model the elastic part of the wagons, and assumes the remainder to be point masses. The parallel graphs of the time-changing action/reaction forces, the velocities and the separation distance of both wagons with respect to the contraction and expansion of linear springs shows the non-linear character of the process which is rather difficult to imagine without modelling. In addition, the physical model when exercised with numerical integration can be a useful preparation for calculus. The model was developed and is presented using the SCHOLA LUDUS teaching, learning and testing principles.

Introduction

Students usually have difficulty classifying collisions into elastic and non-elastic types, and also they do not see the importance of the concept. The usual physics school description of an elastic collision of two bodies is based on the laws of momentum and energy conservation. Two phases of the interaction process are usually qualitatively discussed in technical physics: The 1st phase until the speeds of the colliding bodies become equal, and the 2nd phase until the bodies start to move independently one from the other. Then, time integrals of the forces acting on the interacting bodies are taken into consideration but the magnitude of the forces is usually not exactly known, and hence, the dynamic description of the collision is missing. The goal of this paper is to show the dynamics of the process.

We put an elastic spring as a transition element between the colliding bodies in order to model process development continuously and distinguish its phases. In this way students learn a lot about basic dynamics, modelling [1], [2] and effective uses of computers.

1 Physical model and model simulation

Each of the two colliding bodies (wagons) are modelled by two long linear springs without mass and a point mass. The bodies are in relative motion, but without loss of generality we can consider one wagon initially at rest. During the interaction the velocities of both wagons are changing. The forces of deceleration and/or of acceleration are of the same value given by the spring deformation, but of opposite directions (action and reaction). Using the method of differences and the repetitive index notation of programming we can get a set of equations (1) that models, step-by-step the situation:

$$\left.\begin{array}{lll} t := t + dt & dx := x_2 - x_1 & \\ F := k\,(2L - dx) & a_1 := -F / m_1 & a_2 := F / m_2 \\ v_1 := v_1 + a_1\,dt & v_2 := v_2 + a_2\,dt & \\ x_1 := x_1 + v_1\,dt & x_2 := x_2 + v_2\,dt & \end{array}\right\} \quad (1)$$

where m_1, m_2 are the values of the point masses; v1 v2 their velocities; F the force. dt and dx are small intervals of time and distance; k and L the spring parameters (We assume that each wagon has a spring of the same coefficient and length).

Note: The starting kinetic energy, $W_{10} = m_1 v_{10}^2 / 2$ of the first wagon is restricted by the maximum potential energy of the spring, $W = k L^2 / 2$:-

$$W_{10} < W. \quad (2)$$

(Otherwise, the mutual distance of the wagons dx would become negative, or the wagons stick together, or one destroys the other.) As simple proof of the model correct functioning is the sum of the potential energy of spring and the kinetic energies of both wagons that must be, equal the kinetic energy of relative motion at the beginning of the process,

$$W_{10} = kx^2/2 + m_1 v_1^2 / 2 + m_2 v_2^2 / 2. \quad (3)$$

2 Playing, testing, learning

2.1 Initial imagination of collisions

Free creation and imagination of possible situations for two colliding bodies (wagons, balls etc.) is a very useful starting phase for student motivation and stimulation to further work with the modelling.

2.2 Step-by step explanation of the mechanism

Successive iteration of the cycles in eqs. (1) enables students to "visualise" the mechanism of the elastic collision development:

a) To see how the spring provides, step by step, the redistribution of motion between the first and the second wagons, i.e. how the work of the first wagon is temporally turned into the potential energy of the spring while the spring is contracting, or turned into kinetic energy while the spring is expanding, and the second wagon is accelerated .
b) To see how the law of action and reaction is applied.

2.3 Recognition of process phases

Students can gain a suitable overview of the whole process if the graphs of particular characteristics (Fig.1) are discussed in parallel. From the time development of the forces $F_1(t)$ and $F_2(t)$ acting on each of the wagons, the wagons speeds $v_1(t)$ and $v_2(t)$, and the distance dx(t) between them (respectively the deformation of the springs), there can be clearly recognised the following phases of development:

a) the phase before collision, the springs are not in contact yet ($F(t) = 0$)
b) the phase in which the deceleration of the first wagon approaches a maximum ($0< F(t) < F_{MAXIMUM}$)
c) the state in which the spring stops contracting, the deceleration of the first wagon is a maximum and the speeds of both wagons are the same ($F= F_{MAXIMUM}$)
d) the reflection phase, in which the first wagon is decelerated at lower values (the spring starts expanding, ($F_{MAXIMUM} > F(t) > 0$)
e) the phase after the collision, the springs are no longer in contact, ($F(t) = 0$).

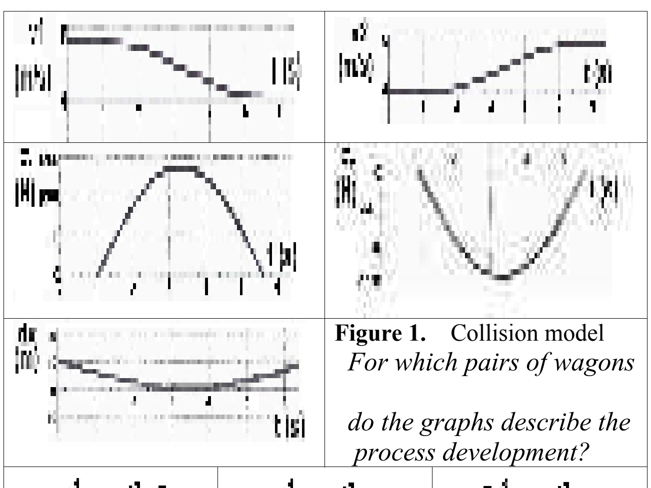

Figure 1. Collision model
For which pairs of wagons do the graphs describe the process development?

2.4 Variability of the model

A good understanding of the process can be gained if students are allowed to play with the model by changing parameters of the model. Five basic qualitatively different cases can be analysed with respect to the masses: the masses of both wagons are equal, or, the mass of the wagon that is initially at rest is smaller, bigger, much smaller or much bigger than the other one. These are the standard cases usually considered for collisions of hard spheres. As m_2 goes to infinity the solution approaches the case of a wall collision.

2.5 Tests for learning

In the frame of SCHOLA LUDUS different cases of the model are supposed for playing, experimenting, learning and teaching by use of tests, similar as in [3].

A test can be provided in the way shown in Figure 1 with the question: *For which pairs of wagons do the graphs describe the process development?* Another graphic task for students can be to draw-in a respective missing graph (without numerical calculation) in a set of parallel graphs, etc. Also, questions involving the dilemma of non-physical solutions can be used. For example, what would happen when the initial kinetic energy is much bigger than the spring potential energy? (Would the first mass bulldozer over the second one, or crush it to smithereens?) Another very important question involves checking that the conversation of energy is maintained by the velocities of the masses - vide eq. (3).

2.5 Validity of the model

In general, models are autonomous. They can be studied and analyzed at will. The essential idea used herein models the deformation forces during elastic collisions as resulting from linear springs obeying Hook's law. This is only a first approximation to non-linear reality. There are also heating effects, friction, drag and irreversible deformation beyond the elastic limit.

All these limitations should be introduced to the students as hints for future work. They beg the question: *How can this model be useful?*, which definitely must be discussed with 16-18 years old students in order to cultivate their critical thinking and modelling skills. For example, consider billiard balls: We can not see the deformation of the balls but the collisions are elastic!

References

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